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High-temperature series analysis of an O(2)-symmetric spin model with discretely valued interaction on 2D lattices

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Abstract. An analysis is given of high-temperature series expansions for the susceptibility of an O(2)-symmetric spin model with discretely valued spin–spin interaction on triangular and square lattices. This analysis does not yield strong evidence for a phase transition in either case, although it is weakly consistent with this possibility. We also comment on the high-temperature series for the free energy and specific heat.

1. Introduction

The critical properties of a statistical model are understood to depend on the space P in which the order parameter lies, the (zero-field) symmetry group G of the Hamiltonian or Euclidean action and the dimensionality, d. At a very general level, one may classify such a model according to whether P and G are discrete or continuous. In commonly studied models, the interaction between the fundamental variables, for example spins, is taken as a continuous function if these variables are continuous and discrete if they are discrete. One may ask what happens if the variables are continuous but the interaction is discrete. In particular, if one takes a given model with continuous P, G and interaction, and changes this interaction to a discretely valued one, does this change the universality class of the model? Introducing a discretely valued interaction in a model with continuous variables yields a far-reaching property of non-zero ground-state disorder. However, this disorder is not necessarily associated with any frustration. What effect does this type of ground-state disorder have on the model?

These questions were first investigated by Guttmann *et al* (1972), Guttmann and Joyce (1973) and Guttmann and Nymeyer (1978). These authors considered an O(2)-symmetric spin model with a (nearest-neighbour) spin–spin interaction of the form $\mathrm{sgn}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j)$. They obtained an exact (zero-field) solution in one dimension which showed that the model remained disordered, with finite correlation length and no long-range order, for all temperatures including $T=0$. They calculated high-temperature series expansion on 3D lattices for the (zero-field) specific heat and, using also a discretised form for the coupling to an external magnetic field, namely $\mathrm{sgn}(\boldsymbol{H} \cdot \boldsymbol{S}_i)$, they calculated high-temperature series expansions for a certain quantity analogous to the susceptibility. From these, they concluded that for dimensionality $d=3$, the model was in the same universality class as the regular O(2)-symmetric spin model with interaction $\boldsymbol{S}_i \cdot \boldsymbol{S}_j$.

Realisations of models with continuous variables but discretely valued interactions (in conjunction with continuously valued interactions) subsequently arose and were studied in the context of topological excitations in 4D U(1) lattice gauge theory (Barber *et al* 1985, Barber and Shrock 1985, Labastida *et al* 1986) and the 3D O(2) model (Kohring *et al* 1986, Kohring and Shrock 1987). The complexity of the discretely valued operators in these models (an integer-valued twelve-angle monopole density operator in the 4D U(1) lattice gauge theory and a four-spin vortex density operator in the 3D O(2) spin model) motivated an investigation of the simplest realisation, namely that in which the discretely valued operator involves only nearest-neighbour spins and takes on only two discrete values.

Thus a further study was carried out by Lee and Shrock (1987, 1988). This included exact solutions of the O(N) version of the sgn spin model for dimensionality $d=1$ and of a modified Gaussian model for arbitrary d with the usual interaction $J\,\Sigma_{\langle ij\rangle}\,\varepsilon_i\varepsilon_j$ of the real-valued site variables ε_n (where $\langle ij\rangle$ denotes nearest-neighbour pairs) replaced by the discretely valued interaction $J\,\Sigma_{\langle ij\rangle}\,\mathrm{sgn}(\varepsilon_i\varepsilon_j)$. In both cases, the model with discretised interaction behaved quite differently from the model with the regular continuous interaction; for example, the modified Gaussian model was in the Ising universality class. For the 3D sgn O(2) model, high-temperature series expansions for the susceptibility were calculated. It was found that the susceptibility exponent γ obtained from an analysis of these expansions on various lattices was consistent with being equal to that for the regular 3D O(2) model and with being equal to the exponent describing the singularity of the function studied by Guttmann and collaborators, so that, in this case, the O(2) models with discrete and continuous interactions were in the same universality class. Analysis of the specific heat series yielded the same conclusion. Finally, Monte Carlo measurements were made of the nearest-neighbour correlation function or internal energy, and of the magnetisation in the model.

The effect of discrete interactions in a model with continuous variables has been more difficult to understand for $d=2$ than for other dimensionalities. The exact result on the modified Gaussian model with discrete interaction (Lee and Shrock 1987) provides an example showing that the universality class can be changed here, as it is in this model for all d. The 2D O(2) model with a $\mathrm{sgn}(\boldsymbol{S}_i\cdot\boldsymbol{S}_j)$ interaction has been studied, beginning with the work of Guttmann and Joyce (1973). The purpose of the present paper is to give high-temperature series expansions for the susceptibility of this model which complement the series of Guttmann and collaborators and to analyse these.

The paper is organised as follows. In § 2 the model is defined and a review is given of previous work. In § 3 the susceptibility series are discussed. Section 4 contains an analysis of these series. Some comments are also given concerning the series for the free energy and specific heat. Section 5 contains our conclusions. In order to render the paper reasonably self-contained, we shall repeat some notation given in our earlier work.

2. The model

The reference model with continuous spin variables and spin–spin interaction is the (classical) O(2) model (also called the plane rotator model), with Hamiltonian

$$\mathscr{H}=-J\sum_{\langle ij\rangle}\boldsymbol{S}_i\cdot\boldsymbol{S}_j-\boldsymbol{H}\cdot\sum_i\boldsymbol{S}_i \tag{2.1}$$

where $S_i = (\cos\theta_i, \sin\theta_i) \in S^1$ and $\langle ij \rangle$ denotes nearest-neighbour pairs of lattice sites i and j. To study the effect of a discrete interaction, a simple and natural choice is to replace the usual scalar product spin–spin interaction by $\mathrm{sgn}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j)$. This yields a model defined by the partition function

$$Z = \int \prod_i \mathrm{d}\Omega_i^{(1)} \,\mathrm{e}^{-\beta\mathscr{H}} \tag{2.2a}$$

where

$$\int \mathrm{d}\Omega^{(1)} \equiv (2\pi)^{-1} \int_{-\pi}^{\pi} \mathrm{d}\theta$$

is the unit-normalised measure on S^1, $\beta \equiv (k_\mathrm{B}T)^{-1}$ and

$$\mathscr{H} = -J \sum_{\langle ij \rangle} \mathrm{sgn}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j) - \boldsymbol{H} \cdot \sum_i \boldsymbol{S}_i. \tag{2.2b}$$

It will be convenient to define $K \equiv \beta J$ and

$$v \equiv \tanh K. \tag{2.3}$$

For the square (sq) lattice, a well known mapping shows that the ferromagnetic ($J > 0$) case of (2.1) is equivalent to the antiferromagnetic case, and similarly for (2.2). In contrast, for triangular (t) lattice, the antiferromagnetic case of (2.1) is frustrated and behaves differently from the ferromagnetic version.

First, one must ask whether for $d = 2$ the model (2.2) has a phase transition at all. If it does, then one must determine whether the universality class of this transition is the same as that of the regular 2D O(2) (plane rotator) model (2.1). The Mermin–Wagner theorem (Mermin and Wagner 1966) forbids spontaneous breaking of the global O(2) symmetry and associated long-range order in this model. Nevertheless, the model does exhibit a phase transition, which has been understood (Kosterlitz and Thouless 1973, Kosterlitz 1974) as being due to the dissociation of vortex pairs as the temperature increases through T_c. In this theory, the free energy and other quantities such as specific heat, susceptibility and correlation length exhibit essential singularities at the critical temperature. Guttmann and Joyce (1973) studied a model with both the spin–spin interaction and the coupling to an external field rendered discrete, namely (2.2a) with

$$\mathscr{H} = -J \sum_{\langle ij \rangle} \mathrm{sgn}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j) - \sum_i \mathrm{sgn}(\boldsymbol{H} \cdot \boldsymbol{S}_i). \tag{2.4}$$

A susceptibility-like function for this model would be

$$1 + 2 \sum_{(ij)} \langle \mathrm{sgn}(\hat{\boldsymbol{H}} \cdot \boldsymbol{S}_i)\, \mathrm{sgn}(\hat{\boldsymbol{H}} \cdot \boldsymbol{S}_j) \rangle \tag{2.5}$$

(where, in our notation, each pair $(ij) = (ji)$ is counted only once in the sum). For reasons of calculational simplicity, instead of analysing this function, Guttmann and collaborators calculated high-temperature series for the function

$$1 + 2 \sum_{(ij)} \langle \mathrm{sgn}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j) \rangle. \tag{2.6}$$

From an analysis of the high-temperature series for (2.6) on the trianglar lattice (Guttmann and Joyce 1973), it was concluded that 'the Padé table does not support a singularity of the assumed form', $\chi(v) \sim A(v_\mathrm{c} - v)^{-\gamma}$. A later study of the model on the trianglar lattice (Guttmann 1978) confirmed this earlier result, finding 'some evidence for a singularity, but no convincing evidence for either an essential or an algebraic singularity.' The same conclusion was reached from a series analysis of the model on the square lattice (Guttmann and Nymeyer 1978) and, by an alternate method, for both the square and triangular lattices (Nymeyer and Guttmann 1985). From a different approach, using approximate Migdal–Kadanoff recursion relations (Barber 1983), it has been asserted that the 2D sgn O(2) model does not have any phase transition at finite temperature, but rather is disordered for all non-zero T. Recently, Monte Carlo simulations have been carried out and have been interpreted first as showing that there is no phase transition (Nymeyer and Irving 1986). However, the opposite conclusion, that the sgn O(2) model does exhibit a phase transition, was reported quite recently (Sánchez-Velasco and Wills 1988) from an extensive Monte Carlo study using finite-size scaling methods.

In earlier work, the present authors calculated and analysed high-temperature series expansions for the susceptibility of the sgn O(2) model on an arbitrary lattice. In view of the current interest in the behaviour of the 2D sgn O(2) model, it seems worthwhile to record our series calculations and analyses for this model. These complement the earlier series work by Guttmann and collaborators since we have calculated the actual susceptibility $\partial M/\partial H$ for the model (2.2) with the usual coupling to the external magnetic field, whereas these earlier authors studied the different function (2.6) for the model (2.4). As will be seen, our series, although of different structure than those of Guttmann and co-workers, yield conclusions in very good agreement with those reached by these authors.

3. High-temperature susceptibility series

It is convenient to define the reduced susceptiblity

$$\bar{\chi} \equiv \beta^{-1}\chi \tag{3.1}$$

which can be calculated via the usual formula

$$\bar{\chi} = 1 + 2\sum_{(ij)} \langle \boldsymbol{S}_i \cdot \boldsymbol{S}_j \rangle. \tag{3.2}$$

As was shown in Lee and Shrock (1987), the natural high-temperature expansion variable for the susceptibility is

$$\bar{v} = 2v/\pi. \tag{3.3}$$

We thus write, for a given lattice Λ,

$$\bar{\chi}_\Lambda = 1 + \sum_{l=1}^{\infty} b_{\Lambda,l}\bar{v}^l. \tag{3.4}$$

As a consequence of a general theorem proved for the O(N) generalisation of (2.2) (Lee and Shrock 1987), it follows that

$$b_{\Lambda,l} = (b_{\Lambda,l})_{\text{Ising}} \qquad \text{for } l < l_{\Lambda,p} \tag{3.5}$$

where l_p denotes the first order where a non-chain graph contributes to χ, and $(b_{\Lambda,l})_{\text{Ising}}$ is the susceptibility coefficient in the high-temperature expansion for $\bar{\chi}$ in the Ising model, namely $(\bar{\chi}_\Lambda)_{\text{Ising}} = 1 + \Sigma_{l=1}^{\infty} (b_{\Lambda,\ l})_{\text{Ising}} v^l$. The $l_{\Lambda,p}$ are

$$l_{\Lambda,p} = \begin{cases} 4 & \text{for } \Lambda = \text{t} \\ 5 & \text{for } \Lambda = \text{sq}. \end{cases} \tag{3.6}$$

Table 1. Susceptibility series coefficients $b_{t,l}$ for the triangular lattice.

n	$b_{t,l}$
1	6
2	30
3	138
4	$618-\frac{3}{4}\pi^3$
5	$2730-3\pi^2-6\pi^3-\frac{5}{8}\pi^4$
6	$11946-\frac{195}{4}\pi^3-5\pi^4-\frac{35}{64}\pi^5$
7	$51882-39\pi^2-\frac{507}{2}\pi^3-\frac{335}{8}\pi^4-\frac{265}{64}\pi^5-\frac{121}{320}\pi^6$

Table 2. Susceptibility coefficients $b_{sq,l}$ for the square lattice.

n	$b_{sq,l}$
1	4
2	12
3	36
4	100
5	$284-\frac{1}{6}\pi^4$
6	$780-\frac{5}{6}\pi^4$
7	$2172+6\pi^2-\frac{25}{6}\pi^4-\frac{7}{120}\pi^6$
8	$5916+12\pi^2-14\pi^4-\frac{4}{15}\pi^6$

We list our results for the series coefficients $b_{\Lambda,l}$ in tables 1 and 2 for the triangular and square lattices, respectively.

Because of the property (3.5), the susceptibility series have the interesting and unusual feature that in low orders they are Ising like (in terms of the variable $\bar{v}$) whereas, as l increases above $l_{\Lambda,p}$, they deviate from the corresponding Ising series and exhibit the true thermodynamic properties of the model (2.2). The susceptibility series coefficients $b_{\Lambda,l}$ are polynomials in π, of degree zero for $l < l_{\Lambda,p}$ and increasing in order and complexity for larger l. They are thus structurally rather different from the coefficients in the high-temperature series expansion of the function (2.6) calculated by Guttmann and Joyce (1973) and Guttmann and Nymeyer (1978), which were integer or rational.

4. Analysis of series

We have analysed the high-temperature susceptibility series (3.4) for the triangular and square lattices using the ratio test, Neville tables and Padé approximants (see Gaunt and Guttmann 1974, Baker 1975). It is appropriate, at the beginning, to examine the series for the simplest form of singularity, namely an algebraic one. If indeed the susceptibility has an essential divergence, then, if fitted to an algebraic form, it would correspond formally to a susceptibility exponent $\gamma = \infty$, since the divergence is more rapid than any power. Thus, for an analysis based on a finite series expansion, a signal of a possible essential divergence would be a value of γ which is unusually large as compared with the typical values which describe spin models.

We thus take

$$\bar{\chi}(\bar{v})_{t,\text{sing}} \sim A(\bar{v})_t(1-\bar{v}/(\bar{v}_c)_t)^{-\gamma} \tag{4.1a}$$

for the close-packed triangular lattice, and

$$\bar{\chi}(\bar{v})_{\text{sq,sing}} \sim A(\bar{v})_{\text{sq}}(1-\bar{v}/(\bar{v}_c)_{\text{sq}})^{-\gamma} + B(\bar{v})_{\text{sq}}(1+\bar{v}/(\bar{v}_c)_{\text{sq}})^{\theta} \tag{4.1b}$$

for the loose-packed square lattice, where $A(\bar{v})_\Lambda$ and $B(\bar{v})_\Lambda$ are analytic. The second term in (4.1*b*) is present because of the above-mentioned mapping between a ferromagnetic and antiferromagnetic version of a spin model on a loose-packed lattice, which implies that, if the free energy is singular at $K = K_c \geqslant 0$, then it it has the same singularity at $K = -K_c$, and similarly with its derivatives.

Using the expansion

$$A(\bar{v})_\Lambda = \sum_{n=0}^{\infty} A_{\Lambda,n}(\bar{v}-\bar{v}_c)^n \tag{4.2a}$$

and defining the critical amplitude as

$$A_\Lambda \equiv A_{\Lambda,0} \tag{4.2b}$$

and similarly for $B(\bar{v})$, the generic form for the dominant singularity of the susceptibility can be expressed as

$$\bar{\chi}(\bar{v})_\Lambda \sim A_\Lambda(1-\bar{v}/(\bar{v}_c)_\Lambda)^{-\gamma}. \tag{4.3}$$

The higher orders of the Padé tables for $\mathrm{d}\ln\bar{\chi}(\bar{v})/\mathrm{d}\bar{v}$ are given in tables 3 and 4 for the triangular and square lattices, respectively. In the $[\mathcal{N}, \mathcal{D}]$ entry in each table the upper number is the pole at $\bar{v} = \bar{v}_c$ and the lower number is minus the residue at this pole, namely the exponent γ.

Table 3. Padé table for $\mathrm{d}\ln\chi(\bar{v})/\mathrm{d}\bar{v}$ for the triangular lattice. In each $[\mathcal{N}, \mathcal{D}]$ entry, the upper number is the pole at $\bar{v} = (\bar{v}_c)_t$ and the lower is the corresponding value of γ. The 'a' indicates an approximant with a spurious, nearly coincident pole-zero pair closer to the origin than the pole listed. The 'b' refers to the fact that for the [1, 5] Padé, there is also a pole at $\bar{v} = 0.389\,01$, which gives rise to a very large value for γ.

	$\mathcal{N}$					
$\mathcal{D}$	0	1	2	3	4	5
1				0.309 22 2.660	0.325 60 3.444	0.333 16 3.952
2			0.309 22 2.660	0.309 30[a] 2.663[a]	0.340 20 4.667	
3		0.323 72 3.449	0.334 99 4.404	0.333 89 ± 0.058 31i —		
4	0.313 29 2.816	0.331 93 4.076	0.312 10[a] 3.028[a]			
5	0.374 45[b] —	0.354 41 ± 0.055 20i —				
6	0.361 88 ± 0.049 98i —					

Table 4. Padé table for $d \ln \chi(\bar{v})/d\bar{v}$ for the square lattice. In each $[\mathcal{N}, \mathcal{D}]$ entry the upper number is the pole at $\bar{v} = (\bar{v}_c)_t$ and the lower is the corresponding value of γ. The notation 'a' has the same meaning as in table 3. A typical complex pair (CP) is the [1, 4] entry, $0.4478 \pm 0.1737i$.

	$\mathcal{N}$						
$\mathcal{D}$	0	1	2	3	4	5	6
1					0.498 10 3.766	0.537 05 5.916	0.405 57 0.828 7
2				0.455 83 2.288	0.564 25 8.668	0.505 57[a] 4.069[a]	
3			0.455 83 2.289	0.455 40[a] 2.279[a]	0.477 18 2.837		
4		CP —	0.493 67 3.707	0.510 55 5.072			
5	0.405 05 1.246	0.460 85 2.288	0.505 99 4.594				
6	CP —	CP —					
7	0.439 94 1.517						

These results do not give firm evidence for any critical singularity of the assumed algebraic form (4.1). From general experience with series expansions for the susceptibility and other quantities, one expects that the series for the close-packed triangular lattice should yield the most stable values for a hypothetical critical point and exponent. As is evident from table 3, however, the Padé approximants do not give stable values for either a critical point (pole in $d \ln \bar{\chi}(\bar{v})/d\bar{v}$) or the associated exponent γ. Indeed, at the highest order calculated, many of the entries, including that for the [3, 3] approximant, do not even yield a pole in $d \ln \bar{\chi}(\bar{v})/d\bar{v}$ on the real-$\bar{v}$ axis, but rather a complex pair of poles. If one were to force this analysis to fit a singularity, then it would weakly indicate $(\bar{v}_c)_t \simeq 0.33 \pm 0.04$ (i.e. $(v_c)_t \simeq 0.52 \pm 0.06$, or $(K_c)_t = 0.58 \pm 0.08$), with the unusually large critical exponent $\gamma \geqslant 2.7$. These findings from our analysis using the actual susceptibility series are in excellent agreement with the conclusions of Joyce and Guttmann (1973), cited above, from their study of the function (2.6). In particular, although these authors concluded that their series did not give convincing evidence for any singularity, they noted that if one were to force a fit to a critical singularity of the form (4.1), then their analysis would give $(v_c)_t \sim 0.5$ and an associated exponent ~ 3.

The Padé approximants for the susceptibility series on the square lattice presented in table 4 are also not highly stable. However, again, if one were to force the approximants to fit the singularity (4.1), then one would conclude that $(\bar{v}_c)_{sq} \simeq 0.49 \pm 0.04$ (i.e. $(v_c)_{sq} \simeq 0.77 \pm 0.06$, $(K_c)_{sq} \simeq 1.0 \pm 0.15$). Again, the lack of any convincing singularity for the model on the square lattice is in agreement with the conclusions reached by Guttmann and Nymeyer (1978) from their different series. It should, however, be noted that the critical value $(K_c)_{sq}$ which we obtained if the assumption of a singularity was forced is consistent, to within the respective uncertainties, with the value $(K_c)_{sq} = 0.91 \pm 0.04$ at which Sánchez-Velasco and Wills (1988) have recently reported evidence for a phase transition from Monte Carlo simulations.

It is also useful to compare these results with the regular 2D O(2) model, to the same order. To do this, we have taken the known high-temperature series expansion for the susceptibility, as a function of K, from Ferer *et al* (1973) and have calculated the dlog Padé in the same manner. As an example, we find the Padé table given as table 5 for the triangular lattice. One sees a substantially more stable value for the critical point and critical exponent, as compared with the results in the corresponding table 3 for the sgn O(2) model on the same lattice. In this context, it is worthwhile to recall that the basic mechanism of vortex dissociation, which drives the phase transition of the usual 2D O(2) model, should not play a significant role in any hypothetical transition in the 2D sgn O(2) model (Guttmann and Nymeyer 1978). This is clear, since one can easily exhibit configurations of spins which would yield a non-zero vortex charge but which have the same internal energy as configurations with zero vortex charge.

In addition to calculating high-temperature series expansions for the susceptibility, we have also calculated such expansions for the free energy and specific heat. For the specific heat per site, C, we write (with $k_B = 1$)

$$C_\Lambda = \sum_{l=2}^{\infty} c_{\Lambda,l} v^l. \tag{4.4}$$

Our results for the square lattice are in complete agreement with those of Guttmann and Nymeyer (1978), which, indeed, extend to higher order. For the triangular lattice, the specific heat series coefficients listed by Guttmann and Joyce (1973) for the 2D

Table 5. Central part of the Padé table for $d \ln \chi(K)/dK$ for the O(2) (plane-rotator) model on the triangular lattice. In each $[\mathcal{N}, \mathcal{D}]$ entry, the upper number is the value $(K_c)_{O(2)}$ of the pole in $d \ln \chi(K)/dK$ and the lower is the corresponding value of γ. The superscript 'aa' indicates two spurious, nearly coincident, pole-zero pairs closer to the origin than the physical pole (and the superscript 'a' has the same meaning as in table 3).

	$\mathcal{N}$						
$\mathcal{D}$	0	1	2	3	4	5	6
1				0.308 411 2.388 5	0.312 181 2.538 1	* *	* *
2			0.308 697 2.400 0	0.305 608[a] 2.319 0[a]	0.305 385[a] 2.315 0[a]	* *	
3		0.310 102[a] 2.449 5[a]	0.309 788[a] 2.440 0[a]	0.305 387[a] 2.315 0[a]	0.305 613[aa] 2.319 1[aa]		
4	0.308 562 2.394 1	0.309 787[a] 2.440 0[a]	0.310 046[aa] 2.448 2[aa]	0.313 314[aa] 2.560 7[aa]			
5	0.315 226 2.711 5	0.302 148[a] 2.234 7[a]	0.313 159[aa] 2.553 8[aa]				
6	* *	* *					
7	* *						

O(2) step model actually refer not to C/k_B but to $(v/K)^2 C/k_B$ (Guttmann 1987). (However, the coefficients listed for comparison for the 2D Ising and regular O(2) models in this reference do refer to C/k_B.) Taking into account this factor, our specific heat series for the triangular (t) lattice agrees with that of Guttmann and Joyce (1973) to the highest order we have checked, namely $O(v^9)$, with the exception of the $O(v^7)$ term, for which we obtain $c_{t,7} = -\frac{541}{20}$, which differs slightly from the value $c_{t,7,GJ} = -\frac{271}{10}$ obtained by Guttmann and Joyce (1973). This difference does not significantly change the conclusions reached from analysis of the specific heat series concerning a possible phase transition in the 2D sgn O(2) model.

5. Conclusions

In summary, we have presented high-temperature series expansions for the susceptibility of the 2D sgn O(2) model and have analysed these to investigate possible critical behaviour. We do not find any strong evidence for a critical singularity, but our Padé tables are weakly consistent with a possible phase transition. Our analysis complements, and is in excellent agreement with, the earlier study by Guttmann and co-workers using high-temperature series expansions for the function (2.6). The subtlety of the effect of switching from the usual continuous $\boldsymbol{S}_i \cdot \boldsymbol{S}_j$ interaction to the $\mathrm{sgn}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j)$ in the 2D O(2) model is not unexpected, since this has the effect, roughly speaking, of increasing the disorder in the model. For $d = 1$, this increase in disorder was sufficient to render the theory disordered and non-critical for all T, including $T = 0$. In contrast, for $d = 3$, although the increase in disorder reduces the magnetisation, it does not remove the phase transition, and, indeed, does not change the universality class of this transition. The case $d = 2$ falls between these dimensionalities and can thus be expected to be more of a borderline situation.

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